# Completing-the-Square for Circles Day 2

***Lesson Objective:* You will use completing-the-square to put circle equations into the center-and-radius form.**

**Summary**: In the last unit, you learned how to write equations for circles. For example, this equation represents a circle centered at (3, –2) with radius 5:

(x – 3)2 + (y + 2)2 = 25

Much less obviously, this equation represents ***the exact same circle***:

x2 – 6x + y2 + 4y = 12

However, that equation hides the information about the center and the radius. To find out the center and the radius, we need to put the equation into the form

**(x – h)2 + (y – k)2 = r2**

The skill needed to do that is one that we’ve used in the previous lesson: ***completing the square*.** Specifically, we need to do that twice; once to make the (*x* – *h*)2 and once to make the (*y* – *k*)2.

**Example:** Put the circle equation x2 – 6x + y2 + 4y = 12 into the center-and-radius form.

First think about how to complete the squares x2 – 6x + \_\_\_\_ and y2 + 4y + \_\_\_\_. The numbers needed are 9 and 4. So add those numbers to both sides of the equation:

x2 – 6x + y2 + 4y = 12  
x2 – 6x ***+ 9*** + y2 + 4y ***+ 4*** = 12 ***+ 9 + 4***  
 (x – 3)2 + (y + 2)2 = 25

**Sometimes you’ll need to rearrange the equation before completing the squares. On the left side, put the *x* terms first, leave a bit of space, the put the *y* terms. Put any plain number terms on the right side. Remember to switch the sign of any term moved to the other side of the equation.**

**Example:** Find the center and radius of the circle x2 + y2 + 32 = 10x + 8y.

First rearrange: x2 – 10x + y2 – 8y = –32  
 Completing-the-squares: x2 – 10x + 25 + y2 – 8y + 16 = –32 + 25 + 16  
 (x – 5)2 + (y – 4)2 = 9

So the center is (5, 4) and the radius is 3.

## Problems (show all work!)

**1.** Each equation below represents a circle. **Use completing the square** to put the equation in center-and-radius form, then identify the center and radius.

**a.** x2 + 8x + y2 – 4y = 5

**b.** x2 – 10x + y2 + 6y – 2 = 0

**c.** 14x + 6y + 22 = – x2 – y2

**d.** x2 + y2 + 12x = 13 **Hint:** only need one completing-the-square here

**2.** Each equation below represents a circle. Find the center and radius, then graph the circle.

|  |  |
| --- | --- |
| **a.** x2 – 4x + y2 + 8y = –11 |  |
| **b.** x2 + 8x + y2 + 18y + 96 = 0 |  |
| **c.** x2 + y2 + 10y + 16 = 0 |  |

**3.** Each equation below represents a circle. **Use completing the square** to put the equation in center-and-radius form, then identify the center and radius.

**a.** x2 + 4x + y2 – 6y = 36

**b.** x2 + y2 + 12x = 12y – 36

**c.** x2 + y2 = 12y + 28

**d.** x2 + y2 = 12x + 28